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$SU(3)$ coupling schemes for odd-odd nuclei in the interacting boson – fermion – fermion model with both odd proton and odd neutron in natural parity orbits

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Abstract. The $SU^{BFF}(3)$ dynamical symmetry limits of interacting boson – fermion – fermion model are identified and they are appropriate for heavy deformed odd – odd nuclei for configurations with both the odd proton and odd neutron occupying all the natural parity orbits in the corresponding valence shells. There are three symmetry limits and their correspondence with two quasi-particle (proton-neutron) Nilsson configurations is established; one of the limits mixes both Nilsson n_z's and Λ's and other two limits mix only Nilsson Λ's. The ^{191}Ir (d,t) ^{190}Ir single nucleon transfer spectroscopic strengths are well described by one of the symmetry limits that mixes only Nilsson Λ's.

PACS. 21.10.Jx Spectroscopic factors – 21.60.Ev Collective models – 21.60.Fw Models based on group theory – 27.80.+w $190 \leq A \leq 219$

1 Introduction

The interacting boson – fermion – fermion model (IBFFM) provides a framework for understanding the structure of quadrupole collective states in heavy odd-odd nuclei. In recent years, with simple IBFFM hamiltonians several numerical studies of odd-odd nuclei are reported ([1–3] and references there in) and there is also progress in developing dynamical symmetry limits of this model associated with the $U(5)$ [4,5], $O(6)$ [6,7], and $SU(3)$ limits of IBM [8,9]; $SU(3)$ limit is appropriate for heavy deformed nuclei. As pointed out in a recent paper [9] for heavy deformed nuclei the orbits occupied by the odd proton and odd neutron can be classified into natural parity orbits (NPO) and intruder parity orbits; in the later case a single j – orbit is involved (see Fig. 1 given ahead). With this there are three types of configurations possible for odd – odd nuclei and denoting proton or neutron, as the case may be, as particle 'a' and the other particle as 'b', the configurations are: (i) $(j_1)^a(j_2)^b$; (ii) $(NPO)^a(j)^b$; (iii) $(NPO_1)^a(NPO_2)^b$. The subscripts '1, 2' indicate that the orbits involved may or may not be the same. Some formal aspects of the band structures associated with $SU(3)$ even – even core coupled to configuration (i) where both odd particles are in single j- orbits are being studied by Paar et al [8]. Recently, the $SU^{BF}(3) \otimes U^F(2j+1)$ dynamical symmetry limit associated with case (ii) is identified and the band structures in this limit are studied by Kota and Pramanik [9]. The purpose of the present paper is to identify the dynamical symmetry limits associated with case (iii), study some of their general features and carry out an application. We will now give a preview.

Section 2 introduces the $SU^{BF_\pi F_\nu}(3)$ limits for $(NPO_1)^a(NPO_2)^b$ configurations coupled to IBM $SU^B(3)$ core. There are three symmetry limits and their correspondence with the Nilsson model is established in Sect. 3. The basis states and energy formula appropriate for one of the three symmetry limits is given in Sect. 4. Sect. 5 deals with single nucleon transfer in the symmetry limits with application to ^{191}Ir (d,t) ^{190}Ir data. Finally Sect. 6 gives some concluding remarks.

2 Coupling schemes with $SU^B(3) \otimes SU^{F_\pi}(3) \otimes SU^{F_\nu}(3)$

With $SU^B(3)$ core, low-lying states of odd-odd nuclei are generated by coupling the two odd particles, odd proton (π) and odd neutron (ν), to the $(2N,0)$ $SU(3)$ irreducible representation (irrep); N is boson number. For the $(NPO_1)^a(NPO_2)^b$ configurations, as can be seen from Fig. 1, the single particle states of the odd proton and odd neutron belong to oscillator quantum numbers η_π and η_ν respectively. Here one assumes, as in the previous IBFM [10] and IBFFM studies [9] that pseudo-spin symmetry is a good symmetry. With this, for rare-earths $\eta_\pi = 3$ and $\eta_\nu = 4$, for actinides $\eta_\pi = 4$ and $\eta_\nu = 5$, for $A \sim 130$ nuclei

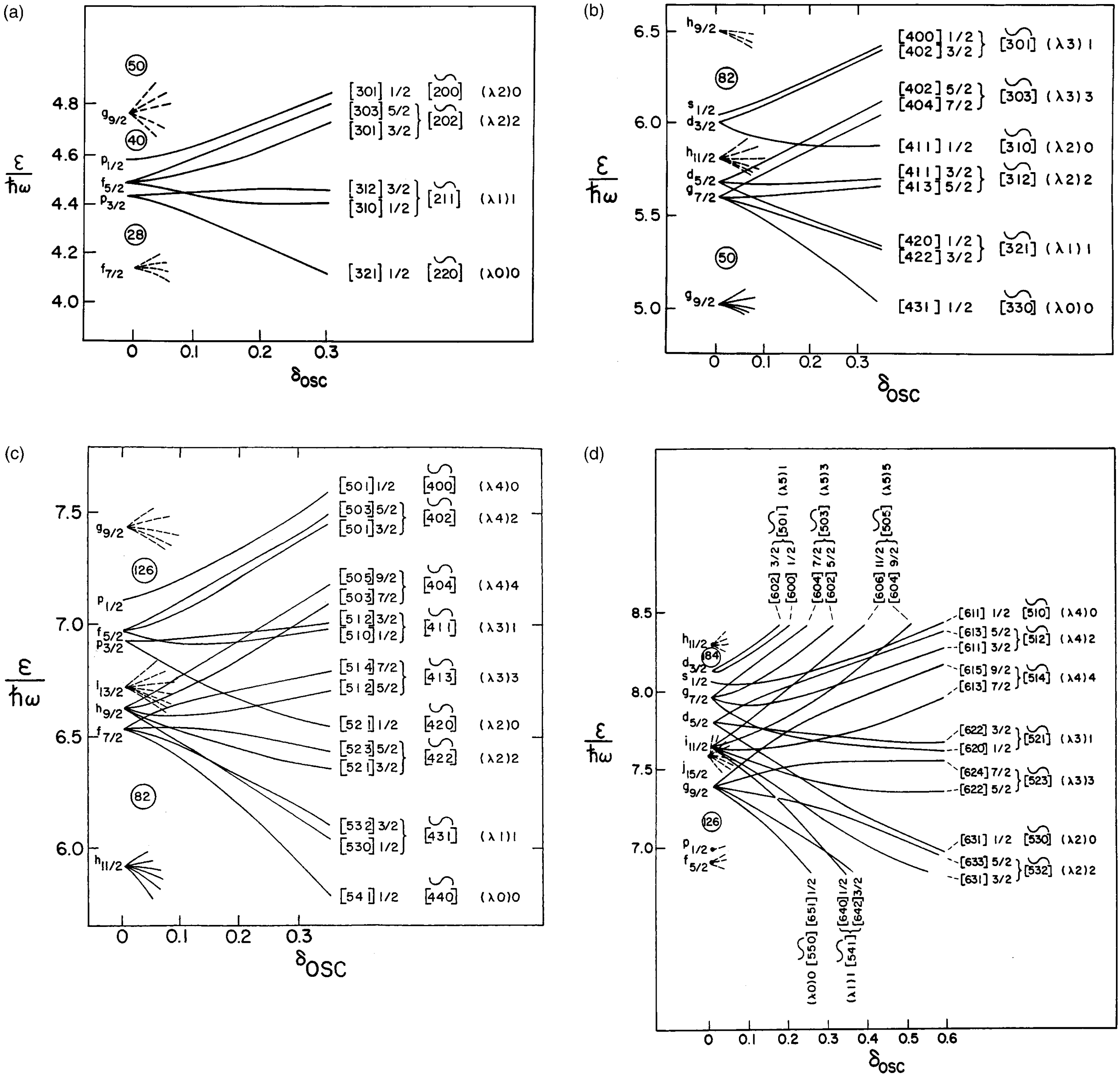

Fig. 1. $SU^{BF}(3)$ limit of IBFM vs Nilsson Model. (a) classification of negitive parity orbits in the 28-40 shell with oscillator shell number $\mathcal{N} = 3$ and $\widetilde{\mathcal{N}} = \eta = 2$. (b) classification of positive parity orbits for $50 < Z < 82$ with oscillator shell number $\mathcal{N} = 4$ and $\widetilde{\mathcal{N}} = \eta = 3$. (c) classification of negative parity orbits for $82 < N < 126$ with oscillator shell number $\mathcal{N} = 5$ and $\widetilde{\mathcal{N}} = \eta = 4$. (d) classification of positive parity orbits for $126 < N < 184$ with oscillator shell number $\mathcal{N} = 6$ and $\widetilde{\mathcal{N}} = \eta = 5$. The Nilsson spectrum for (a) is taken from [15] and for (b), (c) and (d) from [16]. Note that as $N \to \infty$, $\lambda \to 2N$ and for finite N, $\lambda = 2N + \eta - 2\mu$. The correspondence between Nilsson, pseudo Nilsson and $SU^{BF}(3)$ limit labels is described in detail in [10]

$\eta_\pi = 3$ and $\eta_\nu = 3$ etc. There are other prescriptions for defining effective values for η's [11,12] and they are used in the application in Sect. 5. Leaving aside the pseudo-spin (S) generated by $SU(2)$ group, the single odd proton states and odd neutron states belong to the irreps $(\eta_\pi 0)$ and $(\eta_\nu 0)$ of $SU^{F_\pi}(3)$ and $SU^{F_\nu}(3)$ groups respectively [9,10]. Therefore rotational bands with $(NPO_1)^a(NPO_2)^b$ configurations can be classified according to the $SU^{BFF}(3)$ irreps where $SU^B(3) \otimes SU^{F_\pi}(3) \otimes SU^{F_\nu}(3) \supset SU^{BFF}(3)$. Depending on the order of coupling of the $SU(3)$ groups, three coupling schemes are possible (analogous to the $SU(6)$ schemes in [4,13]). The three schemes and the corresponding $SU(3)$ irreps are,

$$\begin{array}{l}\text{I.}\\ SU^B(3) \otimes [SU^{F_\pi}(3) \otimes SU^{F_\nu}(3) \supset SU^{F_\pi F_\nu}(3)] \\ \quad \supset SU^{BFF}(3) \\ |N;(2N,0);(\eta_\pi 0)(\eta_\nu 0)(\lambda_{\pi\nu}\mu_{\pi\nu}); \\ (\lambda_{BFF}\mu_{BFF})K_{BFF}\alpha\rangle \end{array} \tag{1}$$

$$\begin{array}{l}\text{II.}\\ [SU^B(3) \otimes SU^{F_\pi}(3) \supset SU^{BF_\pi}(3)] \otimes SU^{F_\nu}(3) \\ \quad \supset SU^{BFF}(3) \\ |N;(2N,0)(\eta_\pi 0)(\lambda_{BF_\pi}\mu_{BF_\pi}); \\ (\eta_\nu 0);(\lambda_{BFF}\mu_{BFF})K_{BFF}\alpha\rangle \end{array} \tag{2}$$

$$\begin{array}{l}\text{III.}\\ [SU^B(3) \otimes SU^{F_\nu}(3) \supset SU^{BF_\nu}(3)] \otimes SU^{F_\pi}(3) \\ \quad \supset SU^{BFF}(3) \\ |N;(2N,0)(\eta_\nu 0)(\lambda_{BF_\nu}\mu_{BF_\nu}); \\ (\eta_\pi 0);(\lambda_{BFF}\mu_{BFF})K_{BFF}\alpha\rangle \end{array} \tag{3}$$

The labels corresponding to α and the K quantum numbers in (1-3) will be specified later; in many situations the K's in (1-3) are also treated as part of α's. A general hamiltonian that interpolates the symmetry limits $SU^{B-F_\pi F_\nu}(3)$ (I), $SU^{BF_\pi - F_\nu}(3)$ (II) and $SU^{BF_\nu - F_\pi}(3)$ (III) is (with $\hat{C}_2$ denoting $SU(3)$ quadratic Casimir operator),

$$\begin{aligned} H = & \alpha\hat{C}_2\left(SU^B(3)\right) \\ & +\beta\hat{C}_2\left(SU^{BF_\pi}(3)\right) + \gamma\hat{C}_2\left(SU^{BF_\nu}(3)\right) \\ & +\delta\hat{C}_2\left(SU^{F_\pi F_\nu}(3)\right) + \phi\hat{C}_2\left(SU^{BFF}(3)\right) + H' \end{aligned} \tag{4}$$

The structure and role of H' in (4) will be made clear in Sect. 4. For a fixed $SU^B(3)$ irrep $((2N,0))$ and $SU^{BFF}(3)$ irrep $(\lambda_{BFF}\mu_{BFF})$, limit I is obtained for $\beta = \gamma = 0$, limit II for $\gamma = \delta = 0$ and limit III for $\beta = \delta = 0$. The states in I, II and III are related to each other by $SU(3)$ Racah transforms,

$$\begin{aligned} & |N;(2N,0)(\eta_\pi 0)(\lambda_{BF_\pi}\mu_{BF_\pi});(\eta_\nu 0);(\lambda_{BFF}\mu_{BFF})\alpha\rangle_{\text{II}} = \\ & \sum_{(\lambda_{\pi\nu}\mu_{\pi\nu})} U((2N,0)(\eta_\pi 0)(\lambda_{BFF}\mu_{BFF})(\eta_\nu 0); \\ & \qquad (\lambda_{BF_\pi}\mu_{BF_\pi})(\lambda_{\pi\nu}\mu_{\pi\nu})) \times \\ & |N;(2N,0);(\eta_\pi 0)(\eta_\nu 0)(\lambda_{\pi\nu}\mu_{\pi\nu});(\lambda_{BFF}\mu_{BFF})\alpha\rangle_{\text{I}} \end{aligned} \tag{5}$$

$$\begin{aligned} & |N;(2N,0)(\eta_\nu 0)(\lambda_{BF_\nu}\mu_{BF_\nu});(\eta_\pi 0);(\lambda_{BFF}\mu_{BFF})\alpha\rangle_{\text{III}} = \\ & \sum_{(\lambda_{\pi\nu}\mu_{\pi\nu})} (-1)^{\eta_\pi+\eta_\nu+\lambda_{\pi\nu}+\mu_{\pi\nu}} \\ & \qquad U((2N,0)(\eta_\nu 0)(\lambda_{BFF}\mu_{BFF})(\eta_\pi 0); \\ & \qquad (\lambda_{BF_\nu}\mu_{BF_\nu})(\lambda_{\pi\nu}\mu_{\pi\nu})) \times \\ & |N;(2N,0);(\eta_\pi 0)(\eta_\nu 0)(\lambda_{\pi\nu}\mu_{\pi\nu});(\lambda_{BFF}\mu_{BFF})\alpha\rangle_{\text{I}} \end{aligned} \tag{6}$$

In (5,6) the coefficients $U(---)$ are the $SU(3)$ $U-$coefficients [14]. Using the orthonormal properties of the $U-$coefficients in (5,6), it is straight forward to write the states in limit II in terms of those in limit III. The immediate problem to be addressed is about the physical significance of the limits I, II and III. The equivalent question is to find the correspondence between the states (1,2,3) and proton-neutron Nilsson configurations. Now we will turn to this problem.

3 Correspondence with Nilsson model ($N \to \infty$ limit)

3.1 Preliminaries

The IBFM vs Nilsson corresponence shown in Fig. 1, which is the prerequisite for deriving the Nilsson configurations corresponding to the limits I, II and III, follows by considering the $[SU(2) \supset U(1)] \otimes U(1)$ subgroup of $SU(3)$. Here the basis states are $\left|(\lambda\mu)\epsilon\boldsymbol{\Lambda}M_{\boldsymbol{\Lambda}}\right\rangle$ [14],

$$\left|\begin{array}{cccc} SU(3) \supset & [SU(2) \supset U(1)] & \supset U(1) \\ (\lambda,\mu) & \boldsymbol{\Lambda} \quad\quad M_{\boldsymbol{\Lambda}} & \epsilon \end{array}\right\rangle$$

$$\begin{gathered} \epsilon = 2\lambda + \mu - 3(p+q), \boldsymbol{\Lambda} = \mu/2 + (p-q)/2, \\ M_{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda} - 2r \\ 0 \le p \le \lambda, 0 \le q \le \mu, 0 \le r \le 2\boldsymbol{\Lambda} \end{gathered} \tag{7}$$

In (7) $\boldsymbol{\Lambda}$ is a vector as it is a $SU(2)$ label while ϵ and $M_{\boldsymbol{\Lambda}}$ are additive $U(1)$ quantum numbers. The heighest weight $(h.w.)$ state $|(\lambda\mu)h.w.\rangle$ is defined by

$$\begin{gathered} |(\lambda\mu)h.w.\rangle \Longleftrightarrow \left|(\lambda\mu)\epsilon_{max}\boldsymbol{\Lambda}_{max}M_{\boldsymbol{\Lambda}_{max}}\right\rangle \\ \epsilon_{max} = 2\lambda + \mu, \boldsymbol{\Lambda}_{max} = \mu/2, M_{\boldsymbol{\Lambda}_{max}} = \boldsymbol{\Lambda}_{max} \end{gathered} \tag{8}$$

Given that the odd-particle in a odd-A nucleus is in oscillator shell η, the deformed (Nilsson) single particle states $[\mathcal{N}n_z\Lambda]$ correspond to the $\left|(\eta 0)\epsilon_0\boldsymbol{\Lambda}_0M_{\boldsymbol{\Lambda}_0}\right\rangle$ states defined by (7) via the relations,

$$\begin{gathered} [\mathcal{N}n_z\Lambda] \Longleftrightarrow \left|(\eta 0)\epsilon_0\boldsymbol{\Lambda}_0M_{\boldsymbol{\Lambda}_0}\right\rangle \\ \mathcal{N} = \eta, \epsilon_0 = 3n_z - \mathcal{N} \\ \boldsymbol{\Lambda}_0 = (\mathcal{N} - n_z)/2, M_{\boldsymbol{\Lambda}_0} = \Lambda/2. \end{gathered} \tag{9}$$

Note that the Nilsson orbits in (9) and elsewhere in this paper refer to the pseudo Nilsson orbits shown for example in Fig. 1. The relations in (9) follow from (7),

$$\begin{gathered} (\eta 0) \longrightarrow \epsilon_0\boldsymbol{\Lambda}_0M_{\boldsymbol{\Lambda}_0}, \\ \epsilon_0 = 2\eta - 3r, \boldsymbol{\Lambda}_0 = r/2, -\boldsymbol{\Lambda}_0 \le M_{\boldsymbol{\Lambda}_0} \le \boldsymbol{\Lambda}_0; \\ 0 \le r \le \eta \end{gathered} \tag{10}$$

and the fact that ϵ_0 is the quadrupole deformation parameter given by $3n_z - n_x - n_y$ (n_i are oscillator quanta in $i-$th direction; $i = x,y,z$), $2M_{\boldsymbol{\Lambda}_0}$ is projection of angular momentum onto the symmetry axis (i.e $2M_{\boldsymbol{\Lambda}_0}$ is the K-quantum number) and $\boldsymbol{\Lambda}_0 = (n_x + n_y)/2$. It is important to note, as given by (10), that ϵ_0 uniquely defines $\boldsymbol{\Lambda}_0$.

In the $SU^{BF}(3)$ limit of IBFM with $SU^{B}(3)$ core irrep $(2N,0)$ coupled to the $SU^{F}(3)$ irrep $(\eta 0)$ of the odd nucleon, the $SU^{BF}(3)$ irreps $(\lambda_{BF}\mu_{BF})$ are given by (for $N >> \eta$),

$$(2N,0)\otimes(\eta\, 0) \longrightarrow (\lambda_{BF}\mu_{BF}) = \sum_{r=0}^{\eta}(2N+\eta-2r,r) \quad (11)$$

The correspondence between the $SU^{BF}(3)$ limit and Nilsson model shown in Fig. 1 follows from the result that in the asymptotic limit (ASYMP) only $h.w.$ states matter; ASYMP basically correspond to $N \to \infty$ and Sect. 4 gives the other conditions to be satisfied when the laboratory frame $(\lambda\mu)KLM$ quantum numbers are used. With $h.w.$ state for both the N-boson core and the N-boson one-fermion BF systems and using (8,11), the single particle ϵ_0 value is simply given by $\epsilon_0 = \epsilon_{max}\left[(\lambda_{BF}\mu_{BF})\right] - \epsilon_{max}\left[(2N,0)\right] = 2\eta - 3\mu_{BF}$. Then (9) gives the Nilsson orbit that corresponds to the $SU^{BF}(3)$ state $|(2N,0)(\eta 0)\,(\lambda_{BF}\mu_{BF})\,K_{BF}\alpha\rangle$,

$$\begin{gathered}|(2N,0)\,(\eta 0)\;(\lambda_{BF}\,\mu_{BF})\;K_{BF}\,\alpha\rangle_{ASYMP} \Longleftrightarrow \\ [\mathcal{N} n_z \Lambda]\,;\; \mathcal{N}=\eta\,,\; n_z=\eta-\mu_{BF}\,,\; \Lambda=K_{BF}\end{gathered} \quad (12)$$

Alternatively, given a Nilsson orbit $[\mathcal{N} n_z \Lambda]$, the $(\epsilon_0 \boldsymbol{\Lambda}_0 M_{\boldsymbol{\Lambda}_0})$ quantum numbers for the deformed single particle state follow from (9). These in turn generate $h.w.$ states for the BF system (note that $\epsilon^{c}_{max}=4N$, $\boldsymbol{\Lambda}^{c}_{max}=0$ for the core $(2N,0)$ irrep) and hence the $(\lambda_{BF}\mu_{BF})\,K_{BF}$ quantum numbers,

$$\begin{aligned}\mu_{BF} &= \mathcal{N} - n_z \\ \lambda_{BF} &= 2N + \mathcal{N} - 2\mu_{BF} \\ K_{BF} &= \Lambda\end{aligned} \quad (13)$$

Complete discussion of the results in (7-13) is given in [10, 17,18].

3.2 $SU^{B-F_\pi F_\nu}(3)$ limit (I)

The deformed proton - neutron configurations that generate the basis states (1) follow from the results drived [17] in the context of $SU^{BFF}(3)$ limit of IBF^{2}M with two identical particles (two protons or two neutrons) coupled to $SU^{B}(3)$ core $(2N,0)$ irrep. Firstly, the $\pi-\nu$ $SU(3)$ irreps $(\lambda_{\pi\nu}\mu_{\pi\nu})$ generated by $(\eta_\pi\, 0)$ and $(\eta_\nu\, 0)$ are given by,

$$\begin{aligned}&(\eta_\pi\, 0)\otimes(\eta_\nu\, 0) \longrightarrow (\lambda_{\pi\nu}\mu_{\pi\nu}) \\ &= \sum_{r=0}^{\min(\eta_\pi\,,\,\eta_\nu)} (\eta_\pi+\eta_\nu-2r,r)\end{aligned} \quad (14)$$

For example $(30)\otimes(40) \to (\lambda_{\pi\nu}\mu_{\pi\nu}) = (70)\oplus(51)\oplus(32)\oplus(13)$. The $\left(\epsilon_{\pi\nu}\boldsymbol{\Lambda}_{\pi\nu}M_{\boldsymbol{\Lambda}_{\pi\nu}}\right)$ quantum numbers for a given $(\lambda_{\pi\nu}\mu_{\pi\nu})$ follow from (7). The $(\lambda_{BFF}\mu_{BFF})$ irreps arising out of the coupling $(2N,0)\otimes(\lambda_{\pi\nu}\mu_{\pi\nu})$ are generated via the $h.w.$ states of the core irrep and the BFF irrep; $\epsilon^{c}_{max}=4N$, $\boldsymbol{\Lambda}^{c}_{max}=0$, $\epsilon^{max}_{BFF} = 2\lambda_{BFF}+\mu_{BFF} = 4N+\epsilon_{\pi\nu}$, $\boldsymbol{\Lambda}^{max}_{BFF} = \mu_{BFF}/2 = \boldsymbol{\Lambda}_{\pi\nu}$ and $K_{BFF} = 2M_{\boldsymbol{\Lambda}_{\pi\nu}}$. The deformed proton-neutron states $\left|(\eta_\pi 0)(\eta_\nu 0)\,(\lambda_{\pi\nu}\mu_{\pi\nu})\,\epsilon_{\pi\nu}\boldsymbol{\Lambda}_{\pi\nu}M_{\boldsymbol{\Lambda}_{\pi\nu}}\right\rangle$ can be decomposed into deformed single particle states $\left|(\eta_\pi 0)\epsilon_\pi\boldsymbol{\Lambda}_\pi M_{\boldsymbol{\Lambda}_\pi}\right\rangle$ and $\left|(\eta_\nu 0)\epsilon_\nu\boldsymbol{\Lambda}_\nu M_{\boldsymbol{\Lambda}_\nu}\right\rangle$ using $SU(3)\supset SU(2)\otimes U(1)$ reduced Wigner coefficients [14,19] $\langle(\eta_\pi 0)\,\epsilon_\pi\boldsymbol{\Lambda}_\pi\,(\eta_\nu 0)\,\epsilon_\nu\boldsymbol{\Lambda}_\nu \mid (\lambda_{\pi\nu}\mu_{\pi\nu})\,\epsilon_{\pi\nu}\boldsymbol{\Lambda}_{\pi\nu}\rangle$ and the standard Wigner coefficients $\left\langle \boldsymbol{\Lambda}_\pi M_{\boldsymbol{\Lambda}_\pi}\boldsymbol{\Lambda}_\nu M_{\boldsymbol{\Lambda}_\nu} \mid \boldsymbol{\Lambda}_{\pi\nu}M_{\boldsymbol{\Lambda}_{\pi\nu}}\right\rangle$. Putting all these together give the mixure of $\pi-\nu$ Nilsson configuations that correspond to the states (1) of limit I,

$$\begin{aligned}&|N;(2N,0);(\eta_\pi 0)\,(\eta_\nu 0)\,(\lambda_{\pi\nu}\mu_{\pi\nu})\;; \\ &(\lambda_{BFF}\mu_{BFF})\,K_{BFF}\alpha\rangle_{ASYMP} \\ &\Longleftrightarrow \sum_{\epsilon_\pi(\epsilon_\nu),\boldsymbol{\Lambda}_\pi\,,\,\boldsymbol{\Lambda}_\nu\,,M_{\boldsymbol{\Lambda}_\pi}(M_{\boldsymbol{\Lambda}_\nu})} \langle(\eta_\pi 0)\,\epsilon_\pi\boldsymbol{\Lambda}_\pi\,(\eta_\nu 0)\,\epsilon_\nu\boldsymbol{\Lambda}_\nu \mid (\lambda_{\pi\nu}\mu_{\pi\nu})\,\epsilon_{\pi\nu}\boldsymbol{\Lambda}_{\pi\nu}\rangle \times \\ &\left\langle \boldsymbol{\Lambda}_\pi M_{\boldsymbol{\Lambda}_\pi}\boldsymbol{\Lambda}_\nu M_{\boldsymbol{\Lambda}_\nu} \mid \boldsymbol{\Lambda}_{\pi\nu}M_{\boldsymbol{\Lambda}_{\pi\nu}}\right\rangle \times \\ &[\mathcal{N} n_z \Lambda]_\pi\,[\mathcal{N} n_z \Lambda]_\nu\end{aligned} \quad (15)$$

$$\begin{aligned}\epsilon_{\pi\nu} &= 2\lambda_{BFF}+\mu_{BFF}-4N = \epsilon_\pi+\epsilon_\nu \\ \boldsymbol{\Lambda}_{\pi\nu} &= \mu_{BFF}/2;\vec{\boldsymbol{\Lambda}}_{\pi\nu} = \vec{\boldsymbol{\Lambda}}_\pi + \vec{\boldsymbol{\Lambda}}_\nu \\ M_{\boldsymbol{\Lambda}_{\pi\nu}} &= K_{BFF}/2 = M_{\boldsymbol{\Lambda}_\pi} + M_{\boldsymbol{\Lambda}_\nu}\end{aligned} \quad (16)$$

$$\begin{aligned}&\boldsymbol{\Lambda}_\pi = (\eta_\pi - n_{z_\pi})/2 = r_\pi/2, \epsilon_\pi = 3n_{z_\pi} - \eta_\pi = 2\eta_\pi - 3r_\pi; \\ &\eta_\pi = \mathcal{N}_\pi,\; 0\le r_\pi \le \eta_\pi\end{aligned} \quad (17)$$

$$\begin{aligned}&\boldsymbol{\Lambda}_\nu = (\eta_\nu - n_{z_\nu})/2 = r_\nu/2, \epsilon_\nu = 3n_{z_\nu} - \eta_\nu = 2\eta_\nu - 3r_\nu; \\ &\eta_\nu = \mathcal{N}_\nu,\; 0\le r_\nu \le \eta_\nu\end{aligned} \quad (18)$$

Given a $(\lambda_{\pi\nu}\mu_{\pi\nu})$ irrep (using (14)), all the allowed $\epsilon_{\pi\nu}\boldsymbol{\Lambda}_{\pi\nu}$ follow from (7) and then the allowed $(\lambda_{BFF}\mu_{BFF})\,K_{BFF}$ irreps follow from (16); note that $K_{BFF}=\mu_{BFF},\ \mu_{BFF}-2,\ \ldots,\ 0$ or 1 as $\lambda_{BFF} >> \mu_{BFF}$ for $N\to\infty$. This simple pescription for obtaining $(\lambda_{BFF}\mu_{BFF})$ is easily verified to be correct by comparing with more formal Kronecker product formula,

$$\begin{aligned}&(2N,0)\otimes(\lambda_{\pi\nu}\mu_{\pi\nu}) \longrightarrow (\lambda_{BFF}\mu_{BFF}) \\ &= \sum_{a\le\lambda_{\pi\nu},b\le\mu_{\pi\nu}} (2N+\lambda_{\pi\nu}-2a-b,\mu_{\pi\nu}+a-b)\end{aligned} \quad (19)$$

Let us consider the example with $(\lambda_{\pi\nu}\mu_{\pi\nu}) = (32)$. Then $(\epsilon_{\pi\nu},\boldsymbol{\Lambda}_{\pi\nu}) = (8,1), (5,\frac{3}{2}), (5,\frac{1}{2}), (2,2), (2,1), (2,0),$

$(-1,\frac{5}{2})$, $(-1,\frac{3}{2})$, $(-1,\frac{1}{2})$, $(-4,2)$, $(-4,1)$, $(-7,\frac{3}{2})$. The corresponding $(\lambda_{BFF}\mu_{BFF})$ irreps from (16) are,

$$(2N+3,2)\oplus(2N+1,3)\oplus(2N+2,1)\oplus(2N-1,4)\oplus$$
$$(2N,2)\oplus(2N+1,0)\oplus(2N-3,5)\oplus(2N-2,3)\oplus$$
$$(2N-1,1)\oplus(2N-4,4)\oplus(2N-3,2)\oplus(2N-5,3)$$

and (19) gives the same result. The correspondence given by (15) shows that the $(\lambda_{BFF}\mu_{BFF})$ irreps in limit I mix Nilsson configurations with different n_z values. From Fig. 1 it is seen that these configurations are well seperated in energy. Let us consider some examples,

$$\begin{aligned}
&|(2N,0);(30)_\pi(40)_\nu(32)_{\pi\nu};(2N-3,2)K_{BFF}=0\rangle_{ASYMP}\\
&=\sqrt{1/10}\,\{[312\uparrow][402\downarrow]+[312\downarrow][402\uparrow]\}\\
&-\sqrt{2/15}\,\{[310][400]\}\\
&+\sqrt{3/10}\,\{[303\uparrow][413\downarrow]+[303\downarrow][413\uparrow]\}\\
&-\sqrt{1/30}\,\{[301\uparrow][411\downarrow]+[301\downarrow][411\uparrow]\}
\end{aligned}$$

$$\begin{aligned}
&|(2N,0);(30)_\pi(40)_\nu(32)_{\pi\nu};(2N-4,4)K_{BFF}=4\rangle_{ASYMP}\\
&=\sqrt{1/5}\,[312\uparrow][402\uparrow]-\sqrt{2/5}\,[310][404\uparrow]\\
&-\sqrt{1/5}\,[303\uparrow][411\uparrow]+\sqrt{1/5}\,[301\uparrow][413\uparrow]
\end{aligned}$$

$$\begin{aligned}
&|(2N,0);(30)_\pi(40)_\nu(32)_{\pi\nu};(2N-1,1)K_{BFF}=1\rangle_{ASYMP}\\
&=1/3\,[312\uparrow][411\downarrow]+\sqrt{1/3}\,[312\downarrow][413\uparrow]\\
&-\sqrt{2/9}\,[310][411\uparrow]+\sqrt{1/6}\,[303\uparrow][422\downarrow]\\
&-1/3\,[301\uparrow][420\downarrow]+\sqrt{1/18}\,[301\downarrow][412\uparrow]
\end{aligned}\tag{20}$$

In (20) $\uparrow$ implies $\Lambda_{\pi(\nu)}$ is positive and $\downarrow$ implies $\Lambda_{\pi(\nu)}$ is negitive. The $SU(3)\supset SU(2)\otimes U(1)$ reduced Wigner coefficients in (15) are evaluated using Draayer and Akiyama codes [19]. Conventionally the observed rotational bands in heavy ($A\gtrsim 150$) odd-odd nuclei are interpeted as essentially pure two quasi - particle (2 q.p.) Nilsson configurations (see [20,21] and references there in). To the extent this is not a bad approximation, admixing between distant Nilsson orbits (i.e. involving different n_z values) is ruled out. Therefore it is plausible that in real nuclei the $SU^{B-F_\pi F_\nu}(3)$ limit I may not be observed. Now let us turn to limit II (also limit III) states.

3.3 $SU^{BF_\pi - F_\nu}(3)$ limit (II)

In the $SU^{BF_\pi-F_\nu}(3)$ limit (II) $(\lambda_{BF_\pi}\mu_{BF_\pi})$ are good quantum numbers and therefore as argued in Sect. 2.1 the $(\epsilon_\pi\boldsymbol{\Lambda}_\pi)$ are good quantum numbers in the ASYMP limit. Just as in limit I, the $(\lambda_{BFF}\mu_{BFF})\,K_{BFF}$ quantum numbers will fix ϵ^{max}_{BFF}, $\boldsymbol{\Lambda}^{max}_{BFF}$ and $M_{\boldsymbol{\Lambda}_{BFF}}$. As ϵ's are additive, the ϵ_ν for the odd neutron is given by $\epsilon_\nu=\epsilon^{max}_{BFF}-\epsilon_\pi-4N$. The corresponding $\boldsymbol{\Lambda}_\nu$ is unique; see (10). Then it is clear that for the basis states (2), i.e. for fixed $(\lambda_{BF_\pi}\mu_{BF_\pi})$ and $(\lambda_{BFF}\mu_{BFF})$, the single particle n_{z_π} (ϵ_π) and n_{z_ν} (ϵ_ν) are fixed in the ASYMP limit. With this, it is entirely possible that,

$$\begin{aligned}
&|N;(2N,0)\,(\eta_\pi 0)\,(\lambda_{BF_\pi}\mu_{BF_\pi})\,;(\eta_\nu 0)\;;\\
&(\lambda_{BFF}\mu_{BFF})\,K_{BFF}\alpha\rangle_{ASYMP}\\
&\Longleftrightarrow \sum_{M_{\boldsymbol{\Lambda}_\pi}(M_{\boldsymbol{\Lambda}_\nu})}\left\langle \boldsymbol{\Lambda}_\pi M_{\boldsymbol{\Lambda}_\pi}\boldsymbol{\Lambda}_\nu M_{\boldsymbol{\Lambda}_\nu}\mid \boldsymbol{\Lambda}_{BFF}M_{\boldsymbol{\Lambda}_{BFF}}\right\rangle\times\\
&\qquad[\mathcal{N}n_z\Lambda]_\pi\,[\mathcal{N}n_z\Lambda]_\nu
\end{aligned}\tag{21}$$

$$\begin{aligned}
&\epsilon'_{BFF}=2\lambda_{BFF}+\mu_{BFF}-4N,\\
&\boldsymbol{\Lambda}_{BFF}=\mu_{BFF}/2,M_{\boldsymbol{\Lambda}_{BFF}}=K_{BFF}/2\\
&\epsilon_\pi=2\lambda_{BF_\pi}+\mu_{BF_\pi}-4N,n_{z_\pi}=\eta_\pi-\mu_{BF_\pi},\\
&n_{z_\nu}=\left[\eta_\nu+\epsilon'_{BFF}-\epsilon_\pi\right]/3\\
&\Lambda_\pi=2M_{\boldsymbol{\Lambda}_\pi},\Lambda_\nu=2\left(M_{\boldsymbol{\Lambda}_{BFF}}-M_{\boldsymbol{\Lambda}_\pi}\right)\\
&\boldsymbol{\Lambda}_\pi=(\eta_\pi-n_{z_\pi})/2,\boldsymbol{\Lambda}_\nu=(\eta_\nu-n_{z_\nu})/2
\end{aligned}\tag{22}$$

The proof of (21) follows from (5,15), the orthonormal properties of $SU(3)\supset SU(2)\otimes U(1)$ reduced Wigner coefficients and the relation,

$$\begin{aligned}
&U\left((2N,0)\,(\eta_\pi 0)\,(\lambda_{BFF}\mu_{BFF})\,(\eta_\nu 0)\;;\right.\\
&\left.(\lambda_{BF_\pi}\mu_{BF_\pi})\,(\lambda_{\pi\nu}\mu_{\pi\nu})\right)_{ASYMP}\\
&=\langle(\eta_\pi 0)\,\epsilon_\pi\boldsymbol{\Lambda}_\pi\,(\eta_\nu 0)\,\epsilon_\nu\boldsymbol{\Lambda}_\nu\mid(\lambda_{\pi\nu}\mu_{\pi\nu})\,\epsilon_{\pi\nu}\boldsymbol{\Lambda}_{\pi\nu}\rangle\,;\\
&\epsilon_\pi=2\lambda_{BF_\pi}+\mu_{BF_\pi}-4N,\\
&\epsilon_{\pi\nu}=2\lambda_{BFF}+\mu_{BFF}-4N,\epsilon_\nu=\epsilon_{\pi\nu}-\epsilon_\pi\\
&\boldsymbol{\Lambda}_\pi\Longleftrightarrow\epsilon_\pi,;\boldsymbol{\Lambda}_\nu\Longleftrightarrow\epsilon_\nu,\boldsymbol{\Lambda}_{\pi\nu}=\mu_{BFF}/2
\end{aligned}\tag{23}$$

Equation (23) is verified numerically for a variety of values of η_π, η_ν and N and for all allowed $(\lambda\mu)$ irreps in (23). For example the accuracy of (23) is found to be within 2-4% for $(\eta_\pi=\eta_\nu=2,\ N=10,20,40)$ and $(\eta_\pi=3,\ \eta_\nu=4,\ N=10,20)$ cases.

A simple algorithm for generating the basis states (2) and then the expansion (21) in terms of the Nilsson orbits is as follows. Firstly note that n_{z_π} and n_{z_ν} are preserved by the $SU^{BF_\pi-F_\nu}(3)$ limit. Given η_π and η_ν the n_{z_π} and n_{z_ν} are $0\le n_{z_\pi}\le\eta_\pi$ and $0\le n_{z_\nu}\le\eta_\nu$. For a fixed (n_{z_π},n_{z_ν}), the ϵ_π and ϵ_ν are $\epsilon_\pi=3n_{z_\pi}-\eta_\pi=2\eta_\pi-3(\eta_\pi-n_{z_\pi})$ and $\epsilon_\nu=3n_{z_\nu}-\eta_\nu$. Then $\lambda_{BF_\pi}=2N+2n_{z_\pi}-\eta_\pi$ and $\mu_{BF_\pi}=(\eta_\pi-n_{z_\pi})$; also $\boldsymbol{\Lambda}_\pi=\mu_{BF_\pi}/2=(\eta_\pi-n_{z_\pi})/2$ and $\boldsymbol{\Lambda}_\nu=(\eta_\nu-n_{z_\nu})/2$. Using the vector identity $\vec{\boldsymbol{\Lambda}}_{BFF}=\vec{\boldsymbol{\Lambda}}_\pi+\vec{\boldsymbol{\Lambda}}_\nu$ and the additive result $\epsilon_{BFF}=\epsilon_\pi+\epsilon_\nu+4N$ will generate $(\lambda_{BFF}\mu_{BFF})$ irreps via (8); $\lambda_{BFF}=(\epsilon_{BFF}-2\boldsymbol{\Lambda}_{BFF})/2$, $\mu_{BFF}=2\boldsymbol{\Lambda}_{BFF}$. Finally $K_{BFF}=\mu_{BFF}$, $\mu_{BFF}-2$, …, 0 or 1. Let us consider an example with $(\eta_\pi,\eta_\nu)=(3,4)$ and say $(n_{z_\pi},n_{z_\nu})=(1,1)$. They give $\epsilon_\pi=0$, $\epsilon_\nu=-1$, $\epsilon_{BFF}=4N-1$, $\boldsymbol{\Lambda}_\pi=1$, $\boldsymbol{\Lambda}_\nu=3/2$ and $\boldsymbol{\Lambda}_{BFF}=5/2,3/2,1/2$.

Then $(\lambda_{BF_\pi}, \mu_{BF_\pi}) = (2N-1, 2)$ and $(\lambda_{BFF}, \mu_{BFF})\, K_{BFF}$ are $(2N-3, 5) K_{BFF} = 5, 3, 1$, $(2N-2, 3) K_{BFF} = 3, 1$ and $(2N-1, 1) K_{BFF} = 1$. Using (21), the Nilsson correspondence for these states is,

$$
\begin{aligned}
&|(2N,0)(30)_\pi (2N-1,2);\ (40)_\nu\,;\\
&(2N-1,1)K_{BFF}=1\rangle_{ASYMP}\\
&\quad = \sqrt{1/2}\,[312\downarrow]\,[413\uparrow] - \sqrt{1/3}\,[310]\,[411\uparrow]\\
&\quad + \sqrt{1/6}\,[312\uparrow]\,[411\downarrow]\\[1em]
&|(2N,0)(30)_\pi (2N-1,2);(40)_\nu\,;\\
&(2N-2,3)K_{BFF}=1\rangle_{ASYMP}\\
&\quad = -\sqrt{2/5}\,[312\downarrow]\,[413\uparrow] - \sqrt{1/15}\,[310]\,[411\uparrow]\\
&\quad + \sqrt{8/15}\,[312\uparrow]\,[411\downarrow]\\[1em]
&|(2N,0)(30)_\pi (2N-1,2);(40)_\nu\,;\\
&(2N-2,3)K_{BFF}=3\rangle_{ASYMP}\\
&\quad = -\sqrt{3/5}\,[310]\,[413\uparrow] + \sqrt{2/5}\,[312\uparrow]\,[411\uparrow] \qquad (24)\\[1em]
&|(2N,0)(30)_\pi (2N-1,2);(40)_\nu\,;\\
&(2N-3,5)K_{BFF}=1\rangle_{ASYMP}\\
&\quad = \sqrt{1/10}\,[312\downarrow]\,[413\uparrow] + \sqrt{3/5}\,[310]\,[411\uparrow]\\
&+ \sqrt{3/10}\,[312\uparrow]\,[411\downarrow]\\[1em]
&|(2N,0)(30)_\pi (2N-1,2);(40)_\nu\,;\\
&(2N-3,5)K_{BFF}=3\rangle_{ASYMP}\\
&\quad = \sqrt{2/5}\,[310]\,[413\uparrow] + \sqrt{3/5}\,[312\uparrow]\,[411\uparrow]\\[1em]
&|(2N,0)(30)_\pi (2N-1,2);(40)_\nu\,;\\
&(2N-3,5)K_{BFF}=5\rangle_{ASYMP}\\
&\quad = [312\uparrow]\,[413\uparrow]
\end{aligned}
$$

As n_z's are good quantum numbers in the $SU^{BF_\pi - F_\nu}(3)$ limit, the admixing of pure 2 q.p. Nilsson configurations is moderate (only Λ's are mixed) in this limit unlike in the $SU^{B-F_\pi F_\nu}(3)$ limit (I) where n_z's are also mixed. Therefore coupling schemes given by (2) may be observed in data; mixing of 2 q.p. Nilsson configurations is seen in data [20,21].

Before turning to the next section, some remarks about the $SU^{BF_\nu - F_\pi}(3)$ limit (III) are in order. From the discussion prior to (24) it is clear that $SU^{BF_\pi - F_\nu}(3)$ limit states not only have fixed $(\lambda_{BF_\pi}\mu_{BF_\pi})$ irreps but also implicitly fixed $(\lambda_{BF_\nu}\mu_{BF_\nu})$ irreps. Therefore the states (II) and (III) are simply related to each other by a phase factor,

$$
\begin{aligned}
&|N;(2N,0)\,(\eta_\pi 0)\,(\lambda_{BF_\pi}\mu_{BF_\pi})\,;(\eta_\nu 0)\,;\\
&(\lambda_{BFF}\mu_{BFF})\,K_{BFF}\alpha\rangle_{ASYMP}\\
&= (-1)^{\boldsymbol{\Lambda}_\pi + \boldsymbol{\Lambda}_\nu - \boldsymbol{\Lambda}_{BFF}}\,|N;(2N,0)\,(\eta_\nu 0)\,(\lambda_{BF_\nu}\mu_{BF_\nu})\,;\\
&\quad (\eta_\pi 0)\,;(\lambda_{BFF}\mu_{BFF})\,\alpha\rangle\,; \qquad (25)\\
&\boldsymbol{\Lambda}_\pi = \mu_{BF_\pi}/2,\ \boldsymbol{\Lambda}_\nu = \mu_{BF_\nu}/2,\ \boldsymbol{\Lambda}_{BFF} = \mu_{BFF}/2\\
&\mu_{BF_\nu} = (2\eta_\nu - 2\lambda_{BFF} - \mu_{BFF} + 2\lambda_{BF_\pi} + \mu_{BF_\pi})/3,\\
&\lambda_{BF_\nu} = 2N + \eta_\nu - 2\mu_{BF_\nu}
\end{aligned}
$$

The $(\lambda_{BF_\nu}\mu_{BF_\nu})$ for the examples in (24) is $(2N-2, 3)$. Thus in the ASYMP ($N \to \infty$) limit there is one-to-one correspondence between the limits II and III but they differ for finite N; the transformation between states in II and III follows easily by combining (5) and (6). By inverting (21), it is possible to write $[\mathcal{N} n_z \Lambda]_\pi\,[\mathcal{N} n_z \Lambda]_\nu$ configurations in terms of limit II states (2). Then it is seen that the 2 q.p. Nilsson configurations are mixures of $SU^{BFF}(3)$ irreps and therefore they will not have $SU^{BFF}(3)$ symmetry,

$$
\begin{aligned}
&[\mathcal{N} n_z \Lambda]_\pi\,[\mathcal{N} n_z \Lambda]_\nu\\
&\sim \sum_{\boldsymbol{\Lambda}_{BFF}} \left\langle \boldsymbol{\Lambda}_\pi M_{\boldsymbol{\Lambda}_\pi} \boldsymbol{\Lambda}_\nu M_{\boldsymbol{\Lambda}_\nu} \mid \boldsymbol{\Lambda}_{BFF} M_{\boldsymbol{\Lambda}_{BFF}} \right\rangle \qquad (26)\\
&|N;(2N,0)\,(\eta_\pi 0)\,(\lambda_{BF_\pi}\mu_{BF_\pi})\,;\\
&(\eta_\nu 0)\,;\ (\lambda_{BFF}\mu_{BFF})\,K_{BFF}\rangle
\end{aligned}
$$

In (26) Λ_π and Λ_ν uniquely define $M_{\boldsymbol{\Lambda}_\pi}$, $M_{\boldsymbol{\Lambda}_\nu}$, $M_{\boldsymbol{\Lambda}_{BFF}}$ and K_{BFF}. Similarly n_z's define μ_{BF_π}, $\boldsymbol{\Lambda}_\pi$ and $\boldsymbol{\Lambda}_\nu$ and there by $\boldsymbol{\Lambda}_{BFF}$ and μ_{BFF}. Note that by diagonalizing $H_{mix} = \alpha \hat{C}_2(SU^{BF_\pi}(3)) + \beta \hat{K}^2(SU^{BF_\pi}(3)) + \gamma \hat{C}_2(SU^{BFF}(3)) + \delta \hat{K}^2(SU^{BFF}(3))$ in the limit II basis (2) gives (26); the $\hat{K}^2$ operators with eigenvalues K^2 are defined ahead in Sect. 4.

3.4 Summary

Deformed odd-odd nuclei, with good pseudo-spin, admit three coupling schemes and they are: (i) pure 2 q.p. Nilsson configurations having good π and ν n_z's and Λ's; (ii) the limit I states (1) that mix both Nilsson n_z's and Λ's; (iii) the limit II states (2) (or limit III states (3)) that mix only Nilsson Λ's. The scheme (i) is well known and IBFFM generates the two new coupling schemes (ii) and (iii), i.e. limits I and II. Starting with the Nilsson orbits simple algorithms for generating the basis states in limits I and II, without using $SU(3)$ multiplications, is given in Sects. 3.2 and 3.3 respectively. Using these algorithms and the pseudo Nilsson orbits in Fig. 1, it is easy to enumerate the basis states (1)-(3) and their expansions in terms of the 2 q.p. Nilsson configrations. Emperical example for limit II is discussed in Sect. 5 and so far no example for limit I is found (here Nilsson n_z's are mixed). Before turning to Sect. 5, first we give the complete basis and energy formula in limit II.

4 Basis states and energy formula in limit II

The $SU^{BFF}(3)$ dynamical symmetry limits (I, II, III) of IBFFM provide direct laboratory frame description of observables unlike the intrinsic frame description by the 2 q.p. Nilsson configurations. To this end one has to employ the $SU(3) \supset O(3) \supset O(2)$ basis states defined by $|(\lambda_{BFF}\mu_{BFF})K_{BFF}\, L_{BFF}\, M_{BFF}\rangle$ and couple them to the pseudo spin S ($S=0$ or 1) of the odd proton - odd neutron system. For a general $|(\lambda\mu)KLM\rangle$, the Elliott [22] or Vergados [23] K in the ASYMP limit ($\lambda \to \infty$, $\lambda >> \mu$, $\lambda >> L$) is the standard K-label. Then, $L = K, K+1, \ldots$ for $K \neq 0$, $L = 0, 2, 4, \ldots$ for $K = 0$ and λ even and $L = 1, 3, 5, \ldots$ for $K = 0$, λ odd. In order to split the K-bands, Naqvi and Draayer [24] derived a $\hat{K}^2(SU(3))$ operator, that preserves angular momentum, using a rotor - $SU(3)$ mapping. In terms of the quadrupole generator Q^2, angular momentum operator $\hat{L}$ and the $SU(3) \supset O(3)$ integrity basis operators $\hat{X}_3$ and $\hat{X}_4$, the $\hat{K}^2(SU(3))$ operator is

$$\begin{aligned}
&\hat{K}^2(SU(3)) = \frac{\lambda_1\lambda_2\hat{L}^2 + \lambda_3\hat{X}_3^a + \hat{X}_4^a}{2\lambda_3^2 + \lambda_1\lambda_2}; \\
&\hat{X}_3 = \left[(L^1 \times Q^2)^1 \times L^1\right]^0, \\
&\hat{X}_4 = \left[(L^1 \times Q^2)^1 \times (Q^2 \times L^1)^1\right]^0 \\
&\hat{X}_3^a = \frac{\sqrt{30}}{6}\hat{X}_3, \hat{X}_4^a = -\frac{5\sqrt{3}}{18}\hat{X}_4 \\
&\lambda_1 = (-\lambda+\mu)/3, \lambda_2 = (-\lambda-2\mu-3)/3, \\
&\lambda_3 = (2\lambda+\mu+3)/3
\end{aligned} \tag{27}$$

In (27) the normalization of the Q^2 operator is such that $Q^2 \cdot Q^2 = 4\hat{C}_2 - 3\hat{L}^2$ and $\left\langle \hat{C}_2 \right\rangle^{(\lambda\mu)} = \lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)$. Using the ASYMP limit expression for the reduced matrix elements of the Q^2 operator [9,18], we explicitly verified that in the ASYMP limit the $\hat{K}^2(SU(3))$ operator defined by (27) is diagonal in the $|(\lambda\mu)KLM\rangle$ basis and its eigenvalues are exactly K^2. Eq. (27) defines $\hat{K}^2$ operator for $SU^{BF_\pi}(3)$, $SU^{BF_\nu}(3)$ and $SU^{BFF}(3)$ groups with eigenvalues in ASYMP limit being $K^2_{BF_\pi}$, $K^2_{BF_\nu}$ and K^2_{BFF}. The $\hat{K}^2(SU^{BFF}(3))$ operator splits the K_{BFF} bands that belong to a given $(\lambda_{BFF}\mu_{BFF})$. However the hamiltonian should also contain terms that will change the positions of $(\lambda_{BFF}\mu_{BFF})$ bands without admixing them [25]. Following the results in [9,10,17,25] a quadrupole - quadrupole (QQ) plus exchange ($EXCH$) interaction $V_{BFF} = \Gamma_{BFF}V_{QQ} + \Lambda_{BFF}V_{Exch}$ for this purpose is constructed and its basic structure was already given in [17]. In the ASYMP limit, just as in odd-A nuclei case [10], the V_{BFF} is diagonal in the $|(\lambda_{BFF}\mu_{BFF})\,K_{BFF}L_{BFF}\rangle$ basis and its eigenvalues are,

$$\begin{aligned}
&\langle V_{BFF}\rangle^{(\lambda_{BFF}\mu_{BFF})K_{BFF}L_{BFF}} \\
&\quad = (2N)\left[\frac{\Gamma_{BFF}}{4}(2x+y) + \frac{\Lambda_{BFF}}{120}(2x+y)^2\right] \\
&x = \lambda_{BFF} - 2N, y = \mu_{BFF}.
\end{aligned} \tag{28}$$

Combining (4), (27) and (28) and adding the $QQ+EXCH$ V_{BF_π} term with strengths Γ_{BF_π} and Λ_{BF_π} respectively for $SU^{BF_\pi}(3)$ [10], the basis states, hamiltonian and energy formula in limit II are,

$$\begin{aligned}
&\text{basis states: } |N;\,(2N,0)\,(\eta_\pi 0)\,(\lambda_{BF_\pi}\mu_{BF_\pi})\,; \\
&\qquad (\eta_\nu 0)\,;(\lambda_{BFF}\mu_{BFF})\,K_{BFF}L_{BFF}SJ_{BFF}M_{BFF}\rangle\,, \\
&H = E_0 + V_{BF_\pi} + V_{BFF} + \alpha\hat{K}^2(SU^{BFF}(3)) \\
&\qquad + \beta\hat{L}^2_{BFF} + \gamma\hat{S}^2 + \delta\hat{J}^2_{BFF}, \\
&E = E_0 + (2N)\left[\frac{\Gamma_{BF_\pi}}{4}(2\eta_\pi - 3\mu_{BF_\pi})\right. \\
&\qquad \left. + \frac{\Lambda_{BF_\pi}}{120}(2\eta_\pi - 3\mu_{BF_\pi})^2\right] \\
&\quad + (2N)\left[\frac{\Gamma_{BFF}}{4}(2\lambda_{BFF} - 4N + \mu_{BFF})\right. \\
&\qquad \left. + \frac{\Lambda_{BFF}}{120}(2\lambda_{BFF} - 4N + \mu_{BFF})^2\right] \\
&\quad + \alpha K^2_{BFF} + \beta L_{BFF}(L_{BFF}+1) \\
&\quad + \gamma S(S+1) + \delta J_{BFF}(J_{BFF}+1)
\end{aligned} \tag{29}$$

In (29) $S = 0, 1$ and $\boldsymbol{J}_{BFF} = \boldsymbol{L}_{BFF} + \boldsymbol{S}$. The $|L_{BFF}\,S\,J_{BFF}\rangle$ states in (29) can be arranged into $\bar{K}_J$ bands where $\bar{K}_J$ is symbolic,

$$\begin{aligned}
&|(\lambda_{BFF}\mu_{BFF})\,K_{BFF}L_{BFF}SJ_{BFF}M_{BFF}\rangle \\
&\quad \Longleftrightarrow \left|(\lambda_{BFF}\mu_{BFF})\,K_{BFF}L_{BFF}S;\bar{K}_J J_{BFF}M_{BFF}\right\rangle
\end{aligned}$$

$$\begin{array}{lll}
S=0 & & \bar{K}_J = K_{BFF}, \\
& & J_{BFF} = L_{BFF} \\
S=1 & & \\
\lambda_{BFF} = \text{any} & K_{BFF} \neq 0 & \bar{K}_J = K_{BFF}-1, \\
& & J_{BFF} = L_{BFF}-1 \\
& & \bar{K}_J = K_{BFF}, \\
& & J_{BFF} = L_{BFF} \\
& & \bar{K}_J = K_{BFF}+1, \\
& & J_{BFF} = L_{BFF}+1 \\
\lambda_{BFF} = \text{even} & K_{BFF} = 0 & \bar{K}_J = 0, \\
& & J_{BFF} = L_{BFF}+1 = 1,3,5,\ldots \\
& & \bar{K}_J = 1, \\
& & J_{BFF} = L_{BFF}-1, L_{BFF} = 1,2,3,4,\ldots \\
\lambda_{BFF} = \text{odd} & K_{BFF} = 0 & \bar{K}_J = 0, \\
& & J_{BFF} = L_{BFF}-1 = 0,2,4,\ldots \\
& & \bar{K}_J = 1, \\
& & J_{BFF} = L_{BFF}, L_{BFF}+1 = 1,2,3,4,\ldots
\end{array} \tag{30}$$

Instead of the $\bar{K}_J$ bands, it is possible to construct good K_J bands by diagonalizing a $\hat{K}_J^2$ operator in the basis in (29). Such an operator is given recently by Naqvi and Draayer [26] and we verified explicitly that in the AYSMP limit (i.e. in the limit of interest in the present study) $\hat{K}_J^2(SU^{BFF}(3))$ eigenvalues are exactly K_J^2. The basis states with good K_J are

$$|N;(2N,0)\,(\eta_\pi 0)\,(\lambda_{BF_\pi}\mu_{BF_\pi})\,;\ (\eta_\nu 0)\,;$$
$$(\lambda_{BFF}\mu_{BFF})\,K_{BFF};\ SK_S;\ K_J J_{BFF} M_{BFF}\rangle$$

$$\begin{array}{lll}
S=1 & & \\
\lambda_{BFF}=\text{any} & K_{BFF}\neq 0 & K_J=K_{BFF}-1, \\
 & & J_{BFF}=K_J,K_J+1,\ldots \\
 & & K_J=K_{BFF}, \\
 & & J_{BFF}=K_J,K_J+1,\ldots \\
 & & K_J=K_{BFF}+1, \\
 & & J_{BFF}=K_J,K_J+1,\ldots \\
\lambda_{BFF}=\text{even} & K_{BFF}=0 & K_J=0, \\
 & & J_{BFF}=1,3,5,\ldots \\
 & & K_J=1, \\
 & & J_{BFF}=1,2,3,4,\ldots \\
\lambda_{BFF}=\text{odd} & K_{BFF}=0 & K_J=0, \\
 & & J_{BFF}=0,2,4,\ldots \\
 & & K_J=1, \\
 & & J_{BFF}=1,2,3,4,\ldots
\end{array} \tag{31}$$

Bands with $S=0$ are not given in (31) as they are same as $|(\lambda_{BFF}\mu_{BFF})\,K_{BFF}L_{BFF}M_{BFF}\rangle$ bands. The explicit form for the expansion coefficients $C_{L_{BFF}}^{K_{BFF};S;K_J J_{BFF}}$ in the expansion of good K_J states (31) in terms of good LSJ states (29) follow for example from Eq. (4) of [27]. The hamiltonian for good K_J basis states (31) is same as the hamiltonian in (29) except that $\beta\hat{K}_J^2(SU^{BFF}(3))$ replaces $\beta\hat{L}_{BFF}^2$. Finally, it should be mentioned that extensions of (29,30,31) for limits I and III are straightforward.

5 Single nucleon transfer in the $SU^{BFF}(3)$ symmetry limits

In odd mass nuclei single nucleon transfer (SNT) strengths provide finger print patterns [16,28] and IBFM $SU^{BF}(3)$ limit is successful in reproducing the pattern seen for example in ^{185}W nucleus [10]. By the same token it is expected that SNT will be a useful tool for testing the $SU^{BFF}(3)$ coupling schemes (wavefunctions) for deformed odd-odd nuclei. There is recent interest in SNT tests of IBFFM and the Nilsson model with new experimental data produced by Garrett and Burke for ^{190,192,194}Ir isotopes [2,7,20,21,29]. In the analysis of this data it is seen that the nucleus ^{194}Ir is better described by IBFFM with $O(6)$ core (compared to Nisson model which assumes axially symmetric rigid core just as in the $SU^{BFF}(3)$ symmetry limits) [2,7] and this is in good accord with the known result that even-even nuclei in $A \simeq 190$ region exhibit γ-softness. However it is seen from the analysis of SNT data involving ^{192}Ir [29] that the structure of some of the low-lying levels in this nucleus are equally well described by both IBFFM (with $O(6)$ core) and Nilsson model. Following this trend, ^{190}Ir data is analyzed in [20,21] using only the Nilsson model. More recently Balodis [30] studied in detail ^{192}Ir (33 levels assigned to 19 rotational bands) and stated that 'if we restrict ourselves to the enegy a few hundreds of keV, ^{192}Ir is satisfactorily described using the comparatively simple Nilsson model'. These results show that: (i) γ - softness of the core in the case of ^{190}Ir will not effect the SNT strengths for the observed low-lying levels; (ii) it is important to consider mixing of 2 q.p. Nilsson configurations in odd-odd nuclei states. Therefore, for ^{190}Ir to a good approximation one can use $SU^B(3)$ core and from the arguments of Sect. 3.3 combined with (ii), it is seen that SNT involving ^{190}Ir will test the applicability of limit II. Section 5.1 gives the formulation for calculating SNT spectroscopic strengths and using this $^{191}\text{Ir} \longrightarrow {}^{190}\text{Ir}$ SNT data is analyzed in Sect. 5.2.

5.1 SNT strengths in limit II

In odd-A to odd-odd nuclei SNT two situations are possible for the change in the boson number (N) and the number of odd fermions (M_ρ): (i) $\Delta N=0$ and $\Delta M_\rho=1$, $\rho=\pi$ or ν; (ii) $\Delta N=1$ and $\Delta M_\rho=1$, $\rho=\pi$ or ν. For $\Delta N=0$ transitions, the transfer operator to lowest order is,

$$P^\dagger_{\ell\frac{1}{2}j;\rho'}=\zeta_{\ell\frac{1}{2}j;\rho'}a^\dagger_{\ell\frac{1}{2}j;\rho'},\rho'=\pi \text{ or } \nu. \tag{32}$$

The operator for $\Delta N=1$ transitions is more complicated (see for example [7]) and this case is not considered in this paper. In (32) $\zeta_{\ell\frac{1}{2}j;\rho'}$ are parameters to be determined from microscopic considerations or by fitting data. Let us assume that the target odd-A nucleus ground state is a good $SU^{BF}(3)$ state and it is denoted by,

$$\begin{aligned}|i\rangle=\big|N;(2N,0)\,(\eta_\rho 0)\,\big(\lambda_{BF_\rho}\mu_{BF_\rho}\big)\\ K_{BF_\rho}L_{BF_\rho};\ \tfrac{1}{2};J_{BF_\rho}M\big\rangle\,;\rho=\pi \text{ or } \nu\end{aligned} \tag{33}$$

Similarly the final odd-odd nucleus states $|f\rangle$ belong to $SU^{BFF}(3)$ limits I, II or III and for example in limit II they are given by (30). Then the spectroscopic strength $S_{j(\ell)}$ for $(\ell\frac{1}{2}j)_{\rho'}$ transfer is,

$$\begin{aligned}&S_{j(\ell)}\,(\alpha_i J_{i;\rho}\longrightarrow \alpha_f J_{f;\rho\rho'}) \\ &=\Big(\zeta_{\ell\frac{1}{2}j;\rho'}\Big)^2\,(2J_i+1)^{-1}\left|\Big\langle \alpha_f J_{f;\rho\rho'}\ ||\ a^\dagger_{\ell\frac{1}{2}j;\rho'}\ ||\ \alpha_i J_{i;\rho}\Big\rangle\right|^2\end{aligned} \tag{34}$$

Using (33) with $\rho=\pi$ for the initial odd-A states $|\alpha_i J_i\rangle$ and the states (30) for the final odd-odd nucleus states $|\alpha_f J_f\rangle$, the formula, with neutron addition to the target, for the spectroscopic strength in limit II is,

$$S_{j(\ell)} = \left(\zeta_{\ell\frac{1}{2}j;\nu}\right)^2 \frac{2J_{BFF}+1}{2J_{BF_\pi}+1}\chi \left\{ \begin{array}{ccc} L_{BF_\pi} & \ell_\nu & L_{BFF} \\ \frac{1}{2} & \frac{1}{2} & S \\ J_{BF_\pi} & j_\nu & J_{BFF} \end{array} \right\}^2 \times$$

$$|\langle (\lambda_{BF_\pi}\mu_{BF_\pi})\, K_{BF_\pi} L_{BF_\pi}\, (\eta_\nu 0)\, \ell_\nu \;||$$

$$(\lambda_{BFF}\mu_{BFF})\, K_{BFF} L_{BFF} \rangle|^2 \tag{35}$$

In (35) $\chi\{--\}$ is $9-j$ coefficient and $\langle -- \,||\, --\rangle$ is $SU(3) \supset O(3)$ reduced Wigner coefficient. Modifications of (35) for (i) odd-neutron target, (ii) for limits I and III and (iii) for good K_J states (31) is straightforward. For example with $\rho' = \pi$ in (32) and $\rho = \nu$ in (33) the spectroscopic strength in limit III is given by (35) with $\pi \leftrightarrow \nu$. Similarly for good K_J bands in limit II, the formula is obtained by combining (35) with the $C_{L_{BFF}}^{K_{BFF};S;K_J J_{BFF}}$ coefficients. Equation (35) is used in analyzing ^{191}Ir $\longrightarrow$ ^{190}Ir SNT spectroscopic strengths deduced from ^{191}Ir (d,t)^{190}Ir experiment by Garrett and Burke [21] and the results are given in the next subsection.

5.2 Anlysis of ^{191}Ir $\longrightarrow$ ^{190}Ir SNT spectroscopic strengths

Low-lying rotational bands of ^{190}Ir are generated by the proton Nilsson orbits $[400]\frac{1}{2}^+$ and $[402]\frac{3}{2}^+$ which form a pseudo Nilsson doublet (see Fig. 1b) and the neutron Nilsson orbits $[510]\frac{1}{2}^-$ and $[512]\frac{3}{2}^-$ which also form a pseudo Nilsson doublet (see Fig. 1c). These orbits correspond to pseudo - oscillator shells $\tilde{\mathcal{N}} = 3$ and $\tilde{\mathcal{N}} = 4$ respectively as shown in Figs. 1b,c. In certain situations it is possible to adopt a much simpler picture as is the case with the Nilsson orbits for ^{190}Ir. To the extent that the NPO $g_{7/2}$ and $d_{5/2}$ orbits for protons in $\mathcal{N} = 4$ shell can be ignored, the $d_{3/2}$ and $s_{1/2}$ orbits can be assigned to a $\eta = 1$ shell [11]. Then the orbits $[400]\frac{1}{2}^+$ and $[402]\frac{3}{2}^+$ map to [101] and $[411]\frac{1}{2}^+$ to [110] orbit of the $\eta = 1$ shell. Following Sect. 3.1, the corresponding $SU^{BF}(3)$ irreps are $(2N-1,1)K_{BF} = 1$ and $(2N+1,0)K_{BF} = 0$. Similarly ignoring the $h_{9/2}$ and $f_{7/2}$ orbits for neutrons in $\mathcal{N} = 5$ shell, the $p_{3/2}$, $f_{5/2}$ and $p_{1/2}$ orbits can be assigned to a $\eta = 2$ shell [12]. Then the orbit $[501]\frac{1}{2}^-$ maps to [200], $[503]\frac{5}{2}^-$ and $[501]\frac{3}{2}^-$ to [202], $[512]\frac{3}{2}^-$ and $[510]\frac{1}{2}^-$ to [211] and $[521]\frac{1}{2}^-$ to [220] orbit of the $\eta = 2$ shell. The corresponding $SU^{BF}(3)$ irreps are $(2N-2,2)K_{BF} = 0$, $(2N-2,2)K_{BF} = 2$, $(2N,1)K_{BF} = 1$ and $(2N+2,0)K_{BF} = 0$. Describing the low-lying states in terms of $\eta_\pi = 1$ and $\eta_\nu = 2$, the ground state of ^{191}Ir in the $SU^{BF}(3)$ limit is (with $N = 8$) $|(16,0)(10)_\pi(15,1)_{BF_\pi}K_{BF_\pi} = 1, L_{BF_\pi} = 1, s = \frac{1}{2}, J_{BF_\pi} = \frac{3}{2}\rangle$ and similarly the low-lying states of ^{190}Ir in the $SU^{BFF}(3)$ limit II are,

$$|(16,0)(10)_\pi(15,1)_{BF_\pi}\,;\; (20)_\nu\,;$$

$$(\lambda_{BFF}\mu_{BFF})\, K_{BFF} L_{BFF};\; S;\; J_{BFF}\rangle\,.$$

The method described in Sect. 3.3 fixes the $(\lambda_{BFF}\mu_{BFF})$ irreps. The proton and neutron Nilsson orbits that make up the low-lying states of ^{190}Ir are already given and they correspond to $[101]_\pi$ and $[211]_\nu$ orbits with $\epsilon_\pi = -1(\boldsymbol{\Lambda}_\pi = 1/2)$ and $\epsilon_\nu = 1(\boldsymbol{\Lambda}_\nu = 1/2)$ respectively. Then $\epsilon_{BFF}^{max} = 4N$ and $\boldsymbol{\Lambda}_{BFF}^{max} = 0,1$. These give $(\lambda_{BFF}\mu_{BFF})\,K_{BFF} = (16,0)0 \oplus (15,2)0,2$. The $\bar{K}_J$ bands generated by these irreps follow from (30) and they are listed in Table 1. In the ^{191}Ir (d,t) ^{190}Ir experiment [21] six bands in ^{190}Ir are populated with $(NPO_1)^a(NPO_2)^b$ configurations and in each case 1-3 levels are seen. The K_J^π bands, the J^π members and their energies and the structure of these levels in terms of limit II quantum numbers are given in Table 1. Choosing the parameters in the energy formula (29) such that the BFF irrep $(16,0)$ is lower than $(15,2)$, states with larger K_{BFF} are lower in energy and similarly $S = 1$ states to be lower than those with $S = 0$ give essentially the same band sequence as seen in data. In the present example, the parameters Γ_{BF_π}, Λ_{BF_π}, Γ_{BFF} and Λ_{BFF} in (29) do not contribute to the excitation energies. Choosing $\alpha = 29.5$ keV, $\beta = 10.65$ keV, $\gamma = -18.97$ keV and $\delta = 14.28$ keV, all the enegies are reproduced within 20 keV except for the three levels that belong to $K_j = 0_1^-$ band which are lower (compared to experiment) by about 200 keV and the one level that belong to the $K_J = 3_1^-$ band which is higer by 200 keV. A better agreement for the $K_J = 0_1^-$ band is obtained by changing the S^2 operator strength γ but at the expense of the position of the 2^- level of 2_2^- band. No attempt is made to add some extra terms to the hamiltonian in (29) so that better agreements for energies can be obtained. Our major interest being in testing the structure of the wavefunctions. Using Table 1 and (35), total spectroscopic strength $S_T = \sum_j S_{j(\ell)}$ is calculated for each level. The results and their comparison with data are shown in Fig. 2. The agreement between the $SU^{BFF}(3)$ limit II results and experiment is seen to be good. It should be stressed that the agreement shown in Fig. 2 is obtained using the $\bar{K}_J$ scheme (30) which uses $L-S$ (with S being pseudo-spin) coupling. Thus the mixing of 2 q.p. Nilsson configurations as given by limit II in ^{190}Ir explains the SNT data.

6 Conclusions

The $SU^{BFF}(3)$ dynamical symmetry limits of IBFFM are identified in this article and they are appropriate for heavy deformed odd - odd nuclei for $(NPO_1)^a(NPO_2)^b$ configurations. There are three symmetry limits and their correspondence with 2 q.p. (proton-neutron) Nilsson configurations is established. It is shown that IBFFM generates two new coupling schemes - limit I states (1) mixing both Nilsson n_z's and Λ's and limit II states (2) (or limit III states (3)) mixing only Nilsson Λ's respectively. This work and the previous studies [8,9] on symmetry limits for $(j)^a(j)^b$ and $(NPO)^a(j)^b$ configurations respectively with $SU^B(3)$ core complete the identification of $SU(3)$ related dynamical symmetries of IBFFM. Analysis of ^{191}Ir (d,t) ^{190}Ir data using $SU^{BFF}(3)$ limit II, given in Sect. 5, should prompt new SNT experiments populating $(NPO)^a(NPO)^b$ type

Table 1. $\bar{K}_J$ bands with $J \leq 4$ for ^{190}Ir in $SU^{BFF}(3)$ limit II. Given in the last two columns are the observed K_J^π bands and energies of the corresponding band members

$(\lambda_{BFF}, \mu_{BFF})$	K_{BFF}	L_{BFF}	S	$\bar{K}_J$	J_{BFF}	K_J^π	Energy (keV)
(16,0)	0	2	1	1	1	1_1^-	25.9
(16,0)	0	2	1	1	2		83
(16,0)	0	4	1	1	3		
(16,0)	0	4	1	1	4		
(16,0)	0	0	0	0	0	0_1^-	183.2
(16,0)	0	2	0	0	2		313.4
(16,0)	0	4	0	0	4		
(16,0)	0	0	1	0	1		173.8
(16,0)	0	2	1	0	3		
(15,2)	2	2	1	3	3	3_1^-	83
(15,2)	2	3	1	3	4		
(15,2)	2	2	1	2	2	2_1^-	220
(15,2)	2	3	1	2	3		347.8
(15,2)	2	4	1	2	4		
(15,2)	2	2	1	1	1	1_2^-	144
(15,2)	2	3	1	1	2		284.9
(15,2)	2	4	1	1	3		
(15,2)	2	5	1	1	4		
(15,2)	2	2	0	2	2	2_2^-	225
(15,2)	2	3	0	2	3		
(15,2)	2	4	0	2	4		
(15,2)	0	1	1	1	1		
(15,2)	0	1	1	1	2		
(15,2)	0	3	1	1	3		
(15,2)	0	3	1	1	4		
(15,2)	0	1	0	0	1		
(15,2)	0	3	0	0	3		
(15,2)	0	1	1	0	0		
(15,2)	0	3	1	0	2		
(15,2)	0	5	1	0	4		

configurations in odd-odd nuclei. The nuclei ^{185}W and ^{187}Os with odd neutron and ^{169}Tm and ^{175}Re with odd proton are known to be good examples for the $SU^{BF}(3)$ limit of IBFM [10,18] and SNT experiments starting with these nuclei and their neighbours will provide additional tests for the $SU^{BFF}(3)$ symmetry limits. Analysis of ^{190}Ir levels clearly showed that the hamiltonian in $SU^{BFF}(3)$ symmetry limits should be studied further. More detalied study of the spectra (hamiltonians), single nucleon transfer strengths (both with $\Delta N = 0$ and $\Delta N = 1$) and other observables including electromagnetic transition stengths etc. in the $SU^{BFF}(3)$ limits will be given in a future publication.

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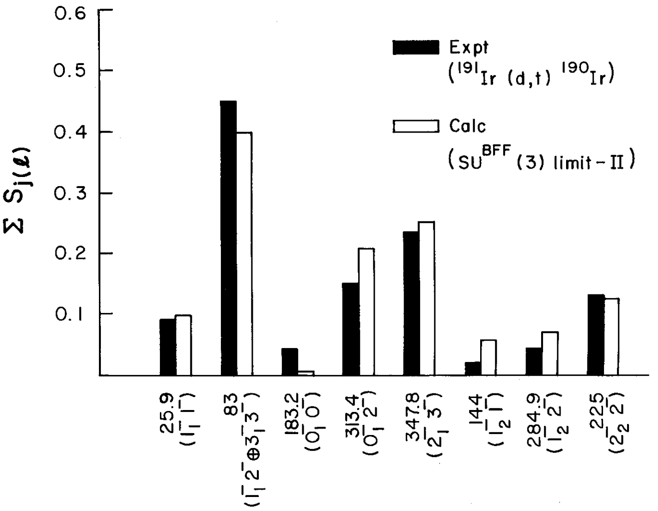

Fig. 2. Total spectroscopic strength $S_T = \sum S_{j(\ell)}$ for ^{190}Ir levels from ^{191}Ir ground state. Data is for ^{191}Ir (d,t) ^{190}Ir reaction. Shown along the x-axis are (K_J^π, J^π) and the corresponding energies in keV. The strength at 83 keV is the summed strength for the two levels shown. The observed strengths to levels at 173.8 keV and 220 keV are not shown as there are doublets at these energies and the (K_J^π, J^π) for only one member is identified. All the data are taken from [21]. Note that error bars for data values are not shown in the figure. The calculations use $SU^{BFF}(3)$ limit II with $\bar{K}_J$ basis (30) and ζ^2 in (35) is chosen to be 0.8. For the eight strengths shown in the figure the corresponding S_T values with good K_J (defined by (31)) are 0.032, 0.494, 0.0, 0.212, 0.071, 0.056, 0.025 and 0.124 respectively. Similarly the S_T values from Nilsson model calculations of [21] are 0.096, 0.457, 0.029, 0.151, 0.209, 0.003, 0.002 and 0.096 respectively

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